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FUNDAMENTALS OF A.C. THEORY

PREVIOUSLY RGP 085

1. INTRODUCTION.

1.1 In Applied Electricity I, we studied the nature of electricity and the related terms and laws. We also examined the components which make up electrical circuits, and studied the characteristics of these circuits under D.C. conditions.

However, to understand the operation of telecommunication equipment, a sound knowledge of alternating current theory must be attained, and this paper outlines the behaviour of resistance, inductance and capacitance in A.C. circuits.

1.2 Although this paper briefly revises information given in Applied Electricity I, before proceeding any further, you should revise the paper "Introduction to A.C. Theory" in Applied Electricity I.

2. ALTERNATING CURRENTS IN TELECOMMUNICATION.

2.1 An Alternating Current is one which regularly alters its direction of flow, first flowing in one direction, then reversing and flowing in the opposite direction. Terms commonly used in connection with A.C. are :-

(i) Sine Wave. A graphical representation of the A.C. produced by a conductor rotating at uniform speed in a uniform magnetic field.

(ii) Cycle. One complete reversal of A.C. passing through a complete set of positive and negative values.

(iii) Frequency. The number of cycles per second. Frequencies used in telecom are expressed in :

- cycles per second (c/s).
- kilocycles per second (kc/s), 1 kc/s = 1,000 c/s.
- megacycles per second (Mc/s), 1 Mc/s = 1,000,000 c/s.

2.2 Alternating currents are used in almost every aspect of telecommunication, ranging from the simple magneto telephone to the complex equipment of radio and television.

The ways in which we use A.C. in telecom are as follows :-

(i) Normal Telephone Conversations use A.C. to carry the message from one telephone to the other, over the telephone line.

(ii) Carrier Telephony uses A.C's. produced within the carrier system as "carriers" for a number of telephone conversations which can be transmitted over one line at the same time. The "carrier" currents have different frequencies which allows them to be separated by filters in the receiving equipment.

(iii) Carrier Telegraphy uses A.C. to provide a number of telegraph channels over one telephone line, or over one channel of a telephone carrier system. The telegraph signals are in the form of pulses of A.C., the frequency of which is different for each channel. As with telephone carrier systems, filters separate the signals at the receiving end, and no interference takes place between messages.

(iv) Radio, which embraces broadcasting, radio telephone and telegraph services and television, uses high frequency alternating currents as carriers for the transmission of signals. We often hear radio stations announce their operating frequency.

2.3 In addition to these actual methods of communication, we use A.C. in other ways. Some of these are :

- to ring telephone bells.
- to provide the supervisory tones in automatic exchanges.
- to "direct-dial" over trunk lines.
- to test the performance of amplifiers and other equipment.
- to operate exchange battery charging equipment.
- to heat, light and ventilate buildings.

A.C.'s. used in telecom vary in wave shape and frequency, and come from many different voltage sources. Some have pure sine wave form, but those used for the transmission of speech and music are more complex. The voltages producing these currents range from a few millivolts in the subscriber's telephone to many kilovolts in a radio transmitter. The frequencies used range from 17 c/s for ringing current up to thousands of megacycles per second in the high frequency radio applications.

2.4 Frequencies Used in Telecom. Frequencies are classified according to their use, and are :-

(i) Power frequency. A.C. at 50 c/s is distributed by Electricity Authorities and provides power for all battery charging equipment and for the normal lighting and power circuits in buildings.

(ii) Audio frequencies are those which produce sounds which can be heard by the human ear. They range from 16 c/s to 20 kc/s, but the limits of hearing vary with different persons. Most people can hear sounds from 30 c/s to 15 kc/s.

It is not necessary to transmit the full audio range for satisfactory sound reproduction. In fact, to do so would be both inconvenient and unnecessary. The range of frequencies we normally use for the electrical transmission of sound is :

- 200 c/s to 3 kc/s for telephone conversations.
- 30 c/s to 7.5 kc/s for programme transmission.

Although there are frequencies in both speech and music which are not present after transmission, the frequencies mentioned above are sufficient for normal reproduction.

(iii) Carrier frequencies are those used in carrier systems for transmitting simultaneous telephone messages over one pair of wires. The frequencies used with open wire line systems range from 3 kc/s to 150 kc/s (approx.). However, carrier systems operating over special cable use frequencies in excess of 1 Mc/s.

(iv) Radio frequencies are those which permit electric current to radiate appreciable electromagnetic energy, and are used for all forms of "wireless" transmission.

Radio Frequencies extend from about 30 kc/s upwards.

Frequencies used in telecom in Australia are shown in Fig. 1.

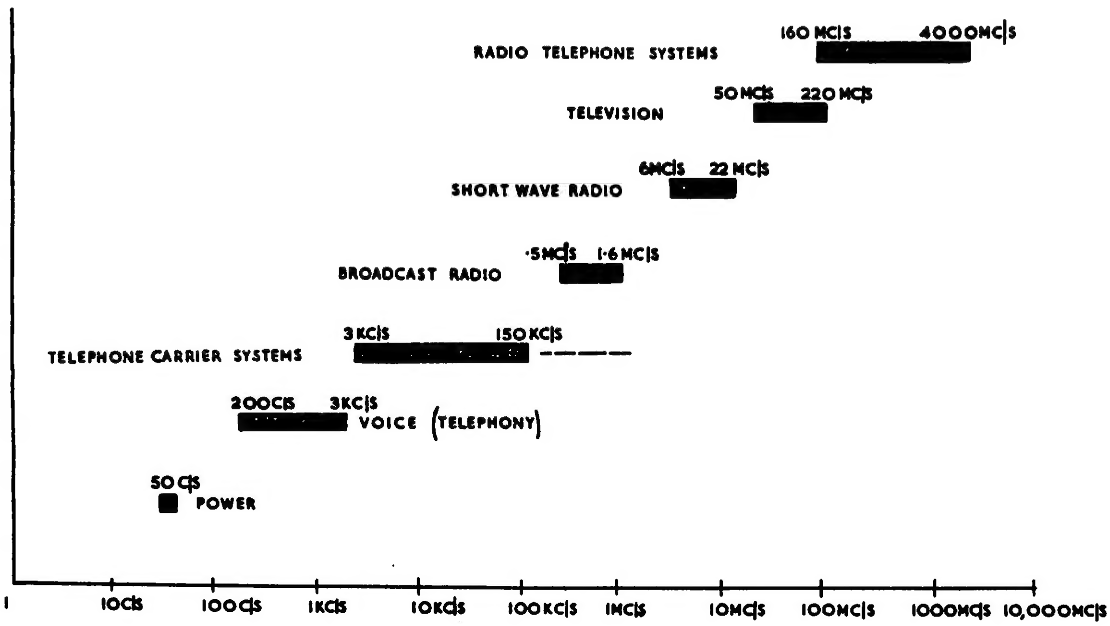


FIG. 1. FREQUENCIES USED IN TELECOM.

3. TRIGONOMETRY AND RIGHT-ANGLE TRIANGLES.

3.1 Trigonometry is a branch of mathematics which deals with the relationships which exist between the sides and angles of triangles.

At this stage it might seem that trigonometry has no connection with A.C., but we will see that currents and voltages in A.C. circuits can be represented by vector diagrams which can be simplified to right-angle triangles. Trigonometry is one of our most useful aids in solving A.C. problems.

3.2 Right-Angle Triangles. We know from earlier studies, the sum of the angles included in a right-angle triangle equals 180°. As one angle is a right-angle, the remaining two angles together contain 90°

The sides of a right-angle triangle are designated with reference to one of these two included angles, as shown in Fig. 2. It is the usual practice to refer to the "reference" angle as "theta", (θ).

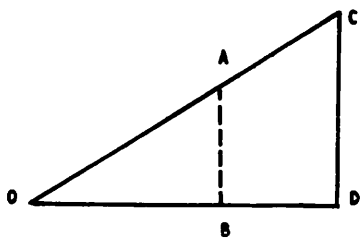
FIG. 2. SIDES OF RIGHT-ANGLE TRIANGLES.

The longest side is the hypotenuse.

The side opposite the reference angle is the opposite side.

The side, which, with the hypotenuse, forms the angle is the adjacent side.

3.3 Trigonometrical Ratios. The ratio between the lengths of the sides of right-angle triangles which have similar reference angles, always works out to the same numerical value, irrespective of the size of the triangle, as in Fig. 3.



$$\frac{AB}{AO} = \frac{CD}{CO}$$

$$\frac{BO}{AO} = \frac{DO}{CO}$$

$$\frac{AB}{BO} = \frac{CD}{DO}$$

FIG. 3. TRIGONOMETRICAL RATIOS.

These ratios are known as sine, cosine and tangent, and are expressed as follows :-

$$\text{Sin } \theta = \frac{\text{OPPOSITE}}{\text{HYPOTENUSE}}$$

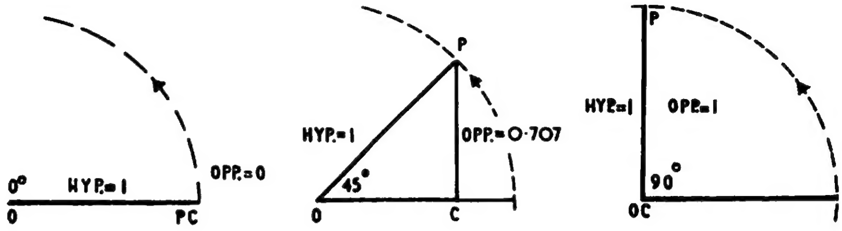
$$\text{Cos. } \theta = \frac{\text{ADJACENT}}{\text{HYPOTENUSE}}$$

$$\text{Tan. } \theta = \frac{\text{OPPOSITE}}{\text{ADJACENT}}$$

3.4 The Numerical Values of sine, cosine and tangent vary according to the magnitude of the reference angle.

For angles between 0° and 90° -

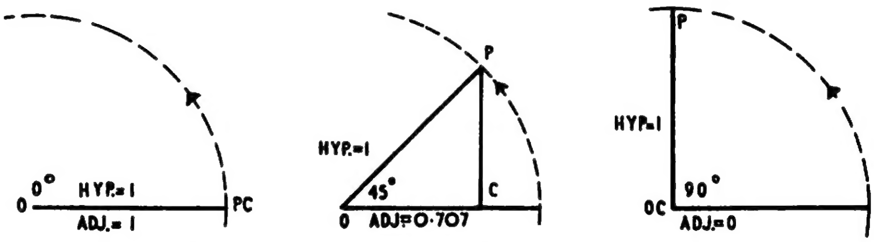
(i) The value of sine changes from zero, at 0°, when the opposite side PC has no length, to unity at 90°, when the opposite side coincides with the hypotenuse, OP. (Figs. 4a, b and c.)



(a) Sine $0^{\circ} = 0$. (b) Sine $45^{\circ} = 0.707$. (c) Sine $90^{\circ} = 1$.

FIG. 4. VALUES FOR SINE.

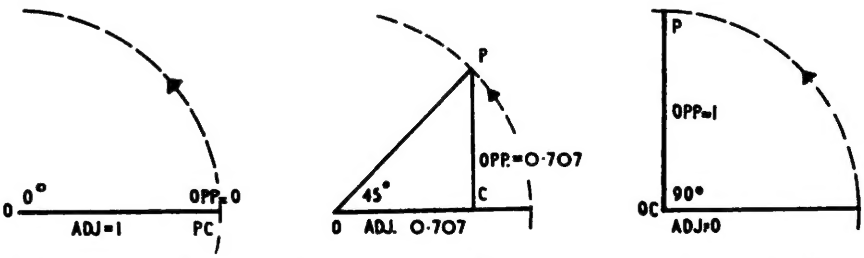
(ii) Cosine has a value of unity at 0° as the adjacent side OC is equal in length to the hypotenuse, OP, but it diminishes in value as the angle θ increases towards 90°. At 90°, cosine is zero as the adjacent side ceases to exist. (Figs. 5a, b and c.)



(a) Cosine $0^{\circ} = 1$. (b) Cosine $45^{\circ} = 0.707$. (c) Cosine $90^{\circ} = 0$.

FIG. 5. VALUES FOR COSINE.

(iii) At 0° tangent has a value of zero, as the opposite side PC has no length. At 45° the opposite side is equal in length to the adjacent side, OC, and tan 45° is unity. After 45° tangent rapidly increases in value until at 90° it is infinite in value, as then the adjacent side has no length. (Figs. 6a, b and c.)



(a) Tangent $0^{\circ} = 0$. (b) Tangent $45^{\circ} = 1$. (c) Tangent 90° = Infinity.

FIG. 6. VALUES FOR TANGENT.

For angles between 90° and 360°, sine, cosine and tangent vary within the same numerical limits as for between 0° and 90°. Whereas sine, cosine and tangent are positive between 0° and 90°, between 90° and 360° there are times when their values are negative.

The familiar sine wave is actually a graph of the variation in values of sine between 0° and 360°.

3.5 Trigonometrical Tables list the numerical values of sine, cosine and tangent for all angles between 0° and 90°. Portion of the "sine" table is shown below.

NATURAL SINES.

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Differences. 1′	2′	3′	4′	5′
0°	0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	14
5	0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	11	14
11	1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	3	6	9	11	14
13	2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	6	11	14
[illegible]	2756	2773	2790	[illegible]	2823	2840	2857	[illegible]	[illegible]	2907	[illegible]	[illegible]	[illegible]	[illegible]	14

SIN 5°

SIN 10° 24′

SIN 15° 32′

ADD

The sine of an angle measured in degrees, or in degrees and certain values of minutes can be read directly from the table of natural sines. For example:-

sin 5° is 0.0872

sin 10° 24' is 0.1805.

To find the sine of an angle which cannot be read directly from the table (for example 15° 32'), the procedure is -

(i) Locate an angle on the table as near as possible to, but less than the required angle, and note its value of sine. (Sin 15° 30' = 0.2672).

(ii) Refer the difference in minutes between the two angles to the "difference" column (2' difference at 15° = 0.0006).

(iii) Add this to the value of sine previously obtained - (0.2672 + 0.0006).

∴ sin 15° 32' = 0.2678.

To find the angle which corresponds to a known value of sine, (for example 0.2323), the procedure is -

(i) On the table, locate an angle whose sine is nearest to, but less than the known sine value, and note its value (sin 13° 24' = 0.2317).

(ii) Refer the difference to the difference column; (0.0006 difference at 13° = 2').

(iii) Add the corresponding difference in minutes to the angle previously noted - (13° 24' + 2' = 13° 26').

∴ 0.2323 = sin 13° 26'

Tables for values of tangent are used in exactly the same way as sine tables, as the values for both sine and tangent increase as the magnitude of the angle increases.

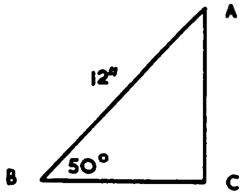
Values of cosine however, decrease as the angle increases, and the numbers shown in the difference column must be subtracted instead of being added.

3.6 Trigonometrical Ratios are used to calculate -

- the remaining two sides of a right-angle triangle when one side and one angle (other than the right angle) are known.
- the included angles of a right-angle triangle when any two sides are known.

Example No.1. In the triangle ABC, calculate -

(i) the length of side BC (ii) the length of side AC.



(i) .... $\cos 50^\circ = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{BC}{AB}$

From tables :

$\cos 50^\circ = 0.6428$

$\therefore 0.6428 = \frac{BC}{12}$

$\therefore BC = 0.6428 \times 12$

$= $ 7.7" (approx.)

(ii) .... $\sin 50^\circ = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{AC}{AB}$

From tables :

$\sin 50^\circ = 0.766$

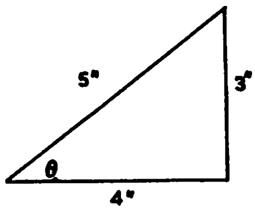
$\therefore 0.766 = \frac{AC}{12}$

$\therefore AC = 0.766 \times 12$

$= $ 9.2" (approx.)

Answer = (i) 7.7" (ii) 9.2".

Example No.2. Calculate the angle θ.



ALTERNATIVE METHODS

$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$= \frac{3}{5}$

$= 0.6$

From tables :

$\sin 36^\circ 52' = 0.6$

$\therefore$ angle $\theta = 36^\circ 52'$

$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$= \frac{4}{5}$

$= 0.8$

From tables :

$\cos 36^\circ 52' = 0.8$

$\therefore$ angle $\theta = 36^\circ 52'$

Answer = $36^\circ 52'$

3.7 There is also a definite relationship between the three sides of a right-angle triangle. This relationship is summarised in Pythagoras' theorem, which states -

"The square on the hypotenuse of a right-angle triangle equals the sum of the squares on the other two sides."

From this we can develop formulas which enable us to calculate any one side of a right-angle triangle, provided the lengths of the other two sides are known. For example, in Fig. 7.

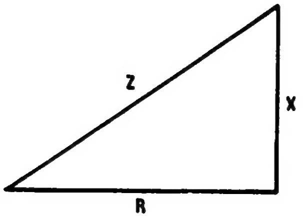


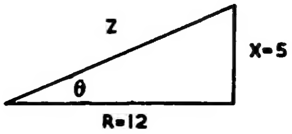
FIG. 7.

$$Z = \sqrt{R^2 + X^2}$$

$$X = \sqrt{Z^2 - R^2}$$

$$R = \sqrt{Z^2 - X^2}$$

Example No.3. In the triangle below, find (i) side Z and (ii) the angle θ.



(i) .... $Z = \sqrt{R^2 + X^2}$

$= \sqrt{12^2 + 5^2}$

$= \sqrt{144 + 25}$

$= \sqrt{169}$

$= \underline{13}$

(ii) ..... $\tan\theta = \dfrac{\text{Opposite}}{\text{Adjacent}}$

$= \dfrac{X}{R} = \dfrac{5}{12}$

$= 0.4166$

From tables -

$\tan 22^\circ 37' = 0.4166$

$\therefore$ Angle $\theta = \underline{22^\circ 37'}$

Answer = (i) 13 units (ii) $22^\circ 37'$.

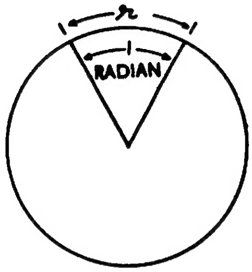
3.8 The calculations involved in Example No.3 are essentially the same as those done when solving A.C. problems. Any one of the three trig. ratios can be used to find the angle, and for practice you should repeat part (ii), using the sin and cos ratios. However, it is the usual practice to use tan in the angle calculation, as the factors involved (X and R) are usually given, or can be calculated with a minimum of working. The factor Z is the denominator in the equations for sin and cos, and as this is nearly always a calculated, and often awkward figure, it is more convenient to use -

$$\text{Tan.}\,\theta = \frac{\text{OPPOSITE}}{\text{ADJACENT}}$$

In the simple aspects of trigonometry dealt with so far, angles have been measured in degrees and minutes. There is another method of angular measurement, called radian measure, which is important in A.C. theory.

3.9 Radian Measure is used to express the angular velocity of, or the angular distance travelled by an object which has rotary motion. When a conductor rotates in a magnetic field to produce sine wave A.C. its rate of rotation can be expressed in terms of radians per second.

A Radian is the angle subtended at the centre of a circle by an arc equal in length to the radius. (Fig. 8).



$$\text{No. of radians in a circle} = \frac{\text{circumference}}{\text{radius}} = \frac{2\pi \times \text{radius}}{\text{radius}} = 2\pi$$

$$\therefore \text{One radian} = \frac{360}{2\pi} = 57.3^\circ.$$

FIG. 8. RADIAN MEASURE.

The rotating conductor traverses 2π radians in making one revolution. When "f" is the number of revolutions per second, the conductor has an angular velocity (symbol ω) or rate of rotation of $2\pi f$ radians per second.

Therefore

$$\omega = 2\pi f$$

where
ω = Angular velocity in radians per second.
2π = 6.28.
f = Revolutions per second.

Example No. 4. Calculate the angular velocity of a conductor which rotates in a magnetic field at 3000 revolutions per minute.

$$\text{No. of revolutions per second} = \frac{\text{Revs. per minute}}{60} = \frac{3000}{60} = 50.$$

$$\text{Angular Velocity } \omega = 2\pi f = 2 \times 3.14 \times 50 = 314 \text{ radians per second}$$

Answer = 314 radians per second.

In A.C. circuits containing inductance or capacitance, the rate of change of current has a direct bearing on the amount of opposition offered by the circuit.

As the rate of rotation of the conductor influences the rate of change of the A.C. produced, we find that wherever the rate of change of current is a factor which governs the behaviour of a circuit, reference is made to the angular velocity of a rotating conductor.

4. VALUES OF ALTERNATING CURRENT AND VOLTAGE.

4.1 As we saw in Applied Electricity I, during each cycle, an A.C. passes through a large range of values. There are several different values associated with A.C. Theory. They are :-

(i) Peak or Maximum Value. This is the maximum value occurring during one cycle. The peak value is not used as a direct indication of the effectiveness of an A.C. as this value is attained at two instants only during each cycle. However we must take peak value into consideration with regard to the insulation of high voltage circuits.

For a sine wave, the relationship between the peak value and effective value is -

PEAK VALUE 1·414 x EFFECTIVE VALUE

(ii) Instantaneous Value. This is the actual value of an A.C., at a certain instant or rotational degree position during a cycle.

For a sine wave -

INSTANTANEOUS VALUE PEAK VALUE x sin. θ

where θ = number of degrees from commencement of cycle.

(iii) Effective or R.M.S. Value. This is the value of A.C. which produces the same heating effect as a continuous current of the same amount. An A.C. of 10 amps (effective) has the same heating effect as 10 amps D.C.

The power of an A.C. is used as a basis for determining its effective value. For one cycle, instantaneous values of current (or voltage) are squared to give proportional values of instantaneous power. These are averaged to obtain a mean value of power for the cycle. A value of current (or voltage) which produces the same power is obtained by taking the square root of the mean power.

This value, (the Root of the Mean of the Squares of instantaneous values) is the effective value of the alternating current or voltage under consideration.

For a sine wave -

EFFECTIVE VALUE 0·707 x PEAK VALUE

Unless definitely stated otherwise, alternating currents and voltages are referred to in terms of effective values. For example, the "240 volt A.C." supply used for home lighting has an effective value of 240 volts. The peak value of this supply is approximately 340 volts. This figure is calculated as follows :-

$$\text{Effective Voltage} = 0.707 \times \text{Peak Voltage.}$$

$$\therefore \quad \text{Peak Voltage} = \frac{\text{Effective Voltage}}{0.707}$$

$$= \text{Effective Voltage} \times 1.414$$

$$= 240 \times 1.414$$

$$= 340 \text{ volts (approx.)}$$

(iv) Average Value. This is the average of all the instantaneous values of the A.C. for one half cycle. Average value is rarely used by technicians, but it has application in electroplating and similar processes.

For a sine wave -

AVERAGE VALUE 0·637 x PEAK VALUE

4.2 Form Factor. This is a number which indicates the wave form of an A.C. It is found from the ratio of the effective value to the average value. Form Factor is used in calibrating the scale of a moving coil meter to read A.C.

$$\textbf{FORM FACTOR} = \frac{\textbf{EFFECTIVE VALUE}}{\textbf{AVERAGE VALUE}}$$

For a sine wave

$$\text{Form Factor} = \frac{0.707 \times \text{Peak Value}}{0.637 \times \text{Peak Value}} = 1.11.$$

As the wave shape becomes flat topped, the form factor tends towards a value of 1.0 When the wave tends to become peaked, the value rises above 1.11.

5. PHASE.

5.1 When an alternating voltage is applied to a circuit, the resulting current has the same frequency as the applied voltage.

In some circuits, the current and voltage waves reach their peak positive or negative values at the same instant. In such circuits, current and voltage are "in phase" (Fig. 9).

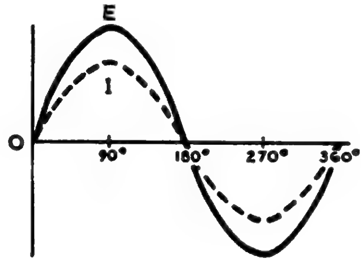
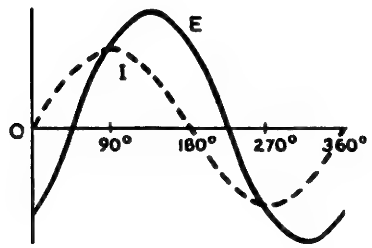


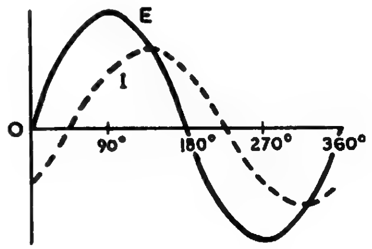
FIG. 9. CURRENT AND VOLTAGE IN PHASE.

In many circuits the current wave reaches its peak value before or after the voltage wave does so. In these cases current and voltage are "out of phase".

5.2 The current in "out of phase" circuits, is said to lead or lag the applied voltage, depending on whether it reaches its maximum positive value before or after the voltage reaches its maximum positive value. (Figs. 10a and b.)



(a) Current Leads Voltage by 45°.



(b) Current Lags Voltage by 45°.

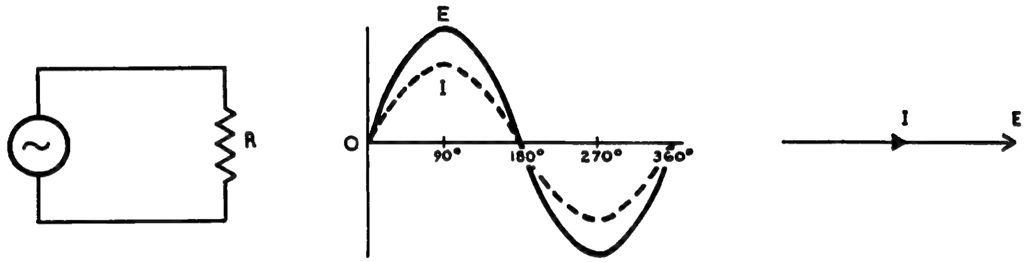
FIG. 10. CURRENT AND VOLTAGE OUT OF PHASE.

★ The time or fraction of a cycle by which the current leads or lags the voltage in an A.C. circuit at a particular frequency is called the phase difference. Phase difference is often expressed as a phase angle (θ) in electrical degrees.

6. RESISTANCE IN A.C. CIRCUITS.

6.1 The Resistance of a material is the opposition it offers to the passage of current, and it depends upon the nature of the material, and upon its length of cross-sectional area. When an A.C. circuit contains resistance only, the current in the circuit can be calculated directly from Ohm's Law, provided that effective values are used.

6.2 Phase Relationship. In purely resistive circuits, (Fig. 11a) changes in the applied voltage cause corresponding changes in the current, with the result that current and voltage are in phase, as in Figs. 11b and c.



(a) (b) (c)

FIG. 11. RESISTANCE IN A.C. CIRCUITS.

6.3 Effect of Frequency. In some A.C. circuits, the resistance of a conductor is found to be higher than when the conductor is part of a D.C. circuit. This is due to the non-uniform distribution of the current over the cross-section of the conductor when A.C. flows.

The passage of A.C. produces an alternating flux which cuts across the conductor developing in it opposing self induced e.m.fs. which are of a higher value at the centre of the conductor than at the surface.

The result is, that current finds an easier path on the outer surface of the conductor, and as such the effective cross-sectional area is reduced.

The tendency for A.C. to flow in the outer "skin" of the conductor in preference to using the whole cross-sectional area is known as "Skin Effect".

Skin Effect is most noticeable at high frequencies, as the higher the frequency the greater are the self induced e.m.fs. at the centre of the conductor, and it is more noticeable in conductors which have a high permeability, for example, in G.I. line wire. It is kept to a minimum by using low permeability conductors that are tubular or which consist of a number of individually insulated interwoven strands.

6.4 Power in Resistive Circuits. When current flows in a resistive circuit it produces heat, irrespective of its direction. As effective values of A.C. produce the same heating effect as similar values of D.C., the rate of energy transformation, or power in a purely resistive circuit can be calculated from -

$$\text{POWER} = E \times I \; : \; I^2 \times R \; : \; \frac{E^2}{R}$$

where

E = Applied Voltage (R.M.S. Value).

I = Current in amperes (R.M.S. Value).

R = Resistance in ohms.

7. INDUCTANCE IN A.C. CIRCUITS.

7.1 An Inductor is a device in which energy is stored in a magnetic field. In the paper "Electromagnetic Induction" of Applied Electricity I, we saw that when the current through an inductor changes, the changing magnetic flux induces an e.m.f. in the circuit, which, according to Lenz's Law, opposes the original change in current.

The ability of a circuit to oppose any change in the current is known as Inductance.

7.2 The Unit of Inductance is the Henry, and a circuit has an inductance of 1 Henry when a current changing at a rate of one ampere per second, induces in it an e.m.f. of one volt.

Whilst the Henry is quite a practical unit, in telecom, we often find it convenient to use submultiple units. They are -

- the millihenry (mH.) which is one thousandth ($\frac{1}{10^3}$ or 10^{-3}) of 1 Henry.
- the microhenry (μH.) which is one millionth ($\frac{1}{10^6}$ or 10^{-6}) of 1 Henry.

All conductors possess inductance; even a straight wire has an e.m.f. induced in it when the magnetic field surrounding it changes. However, almost all practical "inductors" are wound into a coil, the physical properties of which determine the inductance value.

7.3 When A.C. flows in an inductive circuit, the continually changing current produces a continuous opposition or reactance, which is distinct from the normal circuit resistance.

Inductive Reactance (Symbol X_L) is a measure of the opposition to A.C. offered by an inductive circuit. The reactance of a coil is measured in ohms and is determined by -

(i) The Inductance. When the inductance of a coil is increased, the opposition to current change is increased, and so, inductive reactance is increased.

(ii) The Frequency. The magnetic flux, producing the self induced e.m.f. changes at the same rate as the current which produces it. We have seen that the rate of change of current for sine wave A.C. is 2π radians per cycle. When the frequency is increased, the flux changes at a faster rate causing greater opposition.

$$X_L = \omega L$$

where
X_L = Inductive reactance in ohms.
ω = $2\pi f$.
L = Inductance in henries.

In practice, it is impossible to obtain a purely inductive coil, as its windings and connections must possess some resistance, and when this is large, it must be taken into account when determining the total opposition to current. However, when the resistance is small in comparison with the reactance, it can be ignored in simple calculations.

In the inductive components we use in telecom circuits, there are other factors which influence the opposition to A.C. Generally, the total opposition of a component at a certain frequency is reduced when -

- a steady flux is produced in the magnetic circuit by a D.C. in the winding or by the presence of a permanent magnet. This causes partial saturation of the core and prevents the magnetic field from changing to the same degree as the current.
- copper slugs or short-circuit turns are added. The flux caused by the eddy currents in the slugs opposes the main flux and reduces the magnitude of the self-induced e.m.f.

Example No. 5. Find the value of the current when an alternating voltage of 10 volts at frequencies of (i) 1 kc/s, (ii) 2 kc/s is applied to an 80 mH inductor of negligible resistance.

(i)....

$$X_L = \omega L = \frac{6.28 \times 1{,}000 \times 80}{1000} = 500 \text{ ohms (approx.)}$$

$$I = \frac{E}{X_L} = \frac{10 \times 1000}{500} \text{ mA.} = \underline{20 \text{ mA.}}$$

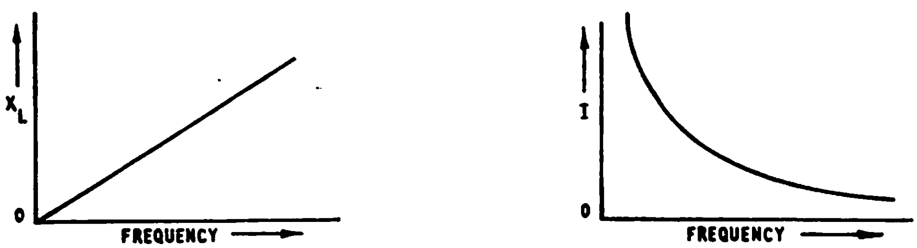
(ii)....

$$X_L = \omega L = \frac{6.28 \times 2{,}000 \times 80}{1000} = 1000 \text{ ohms (approx.)}$$

$$I = \frac{E}{X_L} = \frac{10 \times 1000}{1000} \text{ mA.} = \underline{10 \text{ mA.}}$$

Answer = (i) 20 mA. (ii) 10 mA.

7.4 Effect of Frequency. As shown in Example No. 5, when the frequency of the A.C. applied to an inductor is increased, the inductive reactance is increased and the current is reduced. The variations which occur in reactance and current in a purely inductive circuit when the frequency is varied are as shown in Fig. 12a and b.



(a) Inductive Reactance Increases as Frequency Increases

(b) Current Decreases as Frequency Increases

FIG. 12. EFFECT OF FREQUENCY ON INDUCTIVE CIRCUIT.

7.5 Inductors Connected in Series. When there is no mutual inductance, the total reactance of a number of inductors connected in series is the sum of the individual reactances.

$$X_L = X_{L1} + X_{L2}$$

7.6 Inductors Connected in Parallel. As more than one current path is provided when inductors are connected in parallel, the joint reactance is determined by the same method as is applied to parallel resistors, provided no mutual inductance exists.

$$\frac{1}{X_L} = \frac{1}{X_{L1}} + \frac{1}{X_{L2}}$$

When there is mutual inductance present between series or parallel connected inductors, factors such as the degree of coupling and the method of connection must be considered before the combined reactance can be found.

★ 7.7 Phase Relationship. There is another characteristic of inductive circuits which is important in the study of A.C. The voltage across an inductor reaches its maximum value when the rate of change of current is greatest. In "sine wave" A.C. circuits containing inductance only, this instant occurs when the current is zero value. In such circuits, current lags the applied voltage by 90°.

Fig. 13a illustrates a circuit containing a "pure" inductance (that is, the resistance is negligible). An alternating sine wave voltage is applied, and the resulting current is as shown in Fig. 13b.

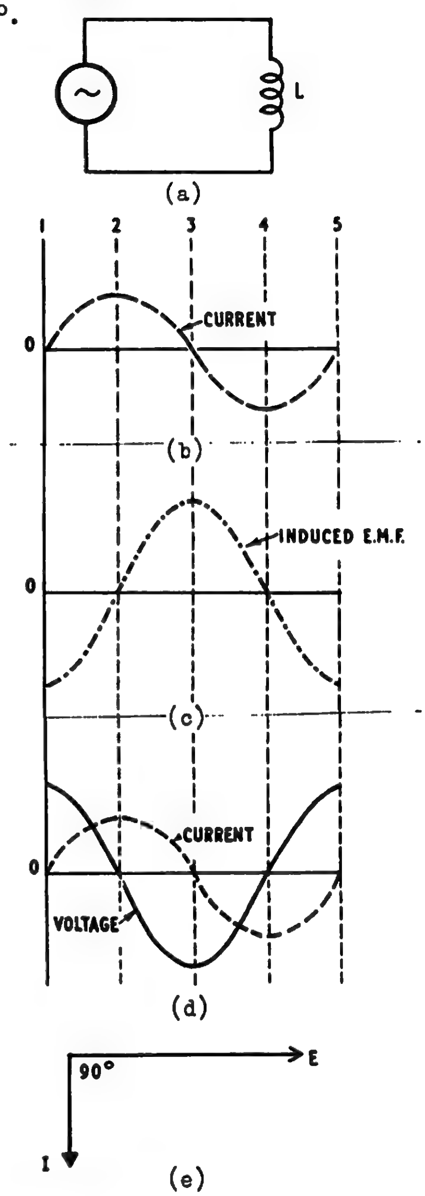
In Fig. 13b we see that at points 2 and 4, the current has reached its maximum value, and as it is momentarily steady, the rate of change of current is zero. Consequently the self induced e.m.f. is zero at these points. (Fig. 13c.)

At points 1, 3 and 5 the current value is zero but here the rate of change of current is the greatest for the cycle. As the magnetic flux has the same rate of change as the current, the rate of change of flux is greatest at these points. The self induced e.m.f. reaches its maximum value when the flux is changing at the maximum rate, that is, at points 1, 3 and 5.

As the self induced e.m.f. opposes the change which produces it, its direction at points 1, 3 and 5 is opposite to the direction of current change at these instants. At points 1 and 5 the current is changing in a positive direction, therefore the self induced e.m.f. is maximum in the negative direction; at point 3 the current is changing in a negative direction, therefore here, the self induced e.m.f. reaches its maximum positive value. This is shown in Fig. 13c.

By Lenz's Law the direction of the induced e.m.f. is at all times opposite to the change which produces it. Here the change is the applied voltage. The applied voltage is therefore, at all instants, equal in magnitude but opposite in direction to the induced e.m.f.

When we draw this voltage in opposition to the self induced e.m.f. (Fig. 13d), we see that in a purely inductive circuit the current reaches its maximum positive value one quarter of a cycle later than the voltage - or the current lags this voltage by 90°. This is shown vectorially in Fig. 13e.



INDUCTANCE IN A.C. CIRCUITS.
FIG. 13.

The statement "current lags the applied voltage by 90°" is correct only when the inductor has no resistance. In practical inductors, which contain resistance as well as inductive reactance, the applied voltage is distributed partly across the resistance and partly across the reactance. In this case, it is only the reactive component of applied voltage, that is, the P.D. across the inductive reactance which is 90° out of phase with current. As the P.D. across the resistance is in phase with current, the current lags the applied voltage (which is the resultant of the P.D's) by less than 90°. This point is dealt with in the paper "Series A.C. Circuits".

7.8 Power in Inductive Circuits. When A.C. flows in an inductor, a magnetic field builds up and collapses every half cycle. During the first quarter cycle, energy is taken from the source to create the magnetic field. This energy is restored to the supply when the field collapses during the next quarter cycle. Although current flows, no energy is expended in a purely inductive circuit.

7.9 Inductance in Telecom Circuits. The applications of inductance in telecom equipment are many and varied, as shown in the following examples.

(i) A.C. Suppression. Rectifiers alone do not produce a steady D.C., but convert each cycle of input A.C. into two half-cycle unidirectional pulses. This is the equivalent of a steady D.C. with a superimposed A.C. component.

When connected in series with the output (Fig. 14) an inductor called a "choke" coil offers high opposition to the A.C. ripple and reduces its magnitude whilst allowing the D.C. to pass readily.

Inductors used in this application range in value from less than 1 henry to approximately 30 henries.

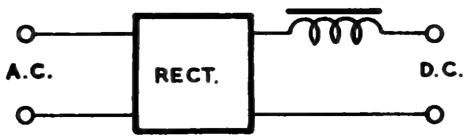


FIG. 14.

Other examples of the application of this characteristic of inductance are found in -

- transmission battery feeds
- C.B. P.M.B.X. alarm (buzzer) circuits.

(ii) Current Control. As the reactance of an inductor is capable of limiting the current in an A.C. circuit without energy loss, variable inductors are employed to control the current in input and output circuits of battery charging equipment.

The reactance of the inductor may be varied in three ways.

- by adjustment of a coil-tapping switch
- by the insertion or withdrawal of a movable portion of the iron core
- by varying the degree of saturation of the magnetic circuit, by varying a D.C. in an auxiliary winding.

This latter method is widely used in modern telecom power plant with automatic output voltage control.

(iii) Loading. In following sections of the paper, dealing with capacitance in A.C. circuits, we will find that the effect of capacitance is opposite to that of inductance, and that one can be used to counter-balance the other.

As the wires in underground cables run side by side, each pair has an appreciable value of capacitance, which adversely affects the transmission performance. By connecting inductors called "loading coils" in series along the cable, the undesirable capacitive effect can be overcome. This process is known as "loading".

Inductance has many other applications in telecom in such items as oscillators, equalizers, filters, aerial coupling units and carrier frequency generators.

These are described in other papers of the course.

8. CAPACITANCE IN A.C. CIRCUITS.

8.1 A Capacitor is a device in which energy is stored in an electric field. In the paper "Capacitance", of Applied Electricity I, we saw that a practical capacitor consists essentially of two conducting plates separated by a thin layer of insulating material, or dielectric.

When an e.m.f. is applied to such a capacitor the electric field produced between the plates distorts the orbits of the outer electrons in the atoms of the dielectric. This limited electron movement in the dielectric, results in a momentary "charging current" in the circuit. When the dielectric is strained in this way, a P.D. is produced across the capacitor which opposes the applied voltage. The charging current ceases when the capacitor counter voltage equals the applied e.m.f. and the capacitor is then said to be "charged".

Should the charging voltage be removed, and the capacitor be connected in a closed circuit, the electrons in the dielectric atoms restore to their normal orbits, and a discharge current, opposite in direction to the charge current, flows in the circuit.

8.2 The Unit of Capacitance is the Farad. A capacitor has a capacitance of one Farad when a charge of one coulomb raises the P.D. across the plates by one volt.

The farad is too large for practical use, and submultiple units are generally used. They are :-

- the microfarad (μF), which is one millionth ($\frac{1}{10^6}$ or 10^{-6}) of one Farad
- the micromicrofarad ($\mu\mu$F) or picofarad (pF), which is one million-millionth ($\frac{1}{10^{12}}$ or 10^{-12}) of one Farad.

8.3 When an alternating e.m.f. is applied to a capacitor, the capacitor alternately charges and discharges. Although no current passes through the dielectric, an A.C. flows in the external circuit. This current is limited by the reactance set up by the capacitor counter voltage.

Capacitive reactance (Symbol X_C) is a measure of the opposition to A.C. offered by a capacitor. The reactance of a capacitor is measured in ohms and is determined by -

(i) The Capacitance. When the capacitance is increased, the quantity of electricity required to charge the capacitor to the same potential is increased, so the current in the circuit must increase also. An increase in current indicates that the capacitive reactance is reduced.

(ii) The Frequency. When the state of charge on the capacitor is changed in any way, current flows in the circuit. When the frequency is increased, the rate of change of charge is increased, which means that the current is correspondingly increased and capacitive reactance is reduced. We have seen, that, for sine wave A.C. the rate of change of current is 2π radians per cycle.

Capacitive reactance can be found from -

$$X_C = \frac{1}{\omega C}$$

where

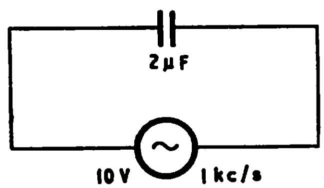
X_C = Capacitive reactance in ohms.

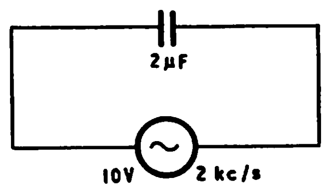
ω = $2\pi f$.

C = Capacitance in Farads.

In practice it is not possible to obtain a purely capacitive capacitor, as its plates and connections must contain some D.C. resistance, but this can be ignored in simple calculations.

Example No. 6. Find the current in the circuit when an alternating voltage of 10 volts at a frequency of (i) 1 kc/s (ii) 2 kc/s is applied to a 2μF capacitor of negligible resistance.





(i) .... $X_C = \frac{1}{\omega C}$

$$= \frac{1,000,000}{6.28 \times 1,000 \times 2}$$

$= 80$ ohms (approx.)

$$I = \frac{E}{X_C}$$

$$= \frac{10 \times 1,000}{80} \text{ mA}$$

$= \underline{125 \text{ mA.}}$

(ii) .... $X_C = \frac{1}{\omega C}$

$$= \frac{1,000,000}{6.28 \times 2,000 \times 2}$$

$= 40$ ohms (approx.)

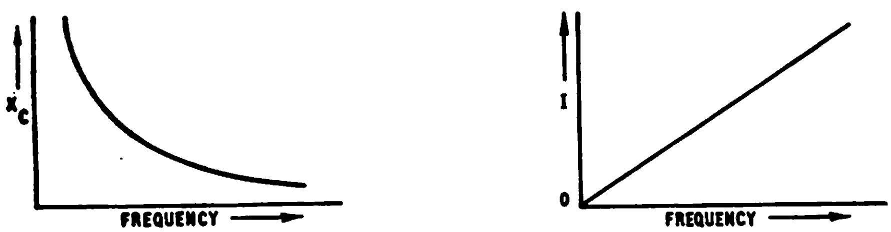
$$I = \frac{E}{X_C}$$

$$= \frac{10 \times 1,000}{40} \text{ mA.}$$

$= \underline{250 \text{ mA.}}$

Answer = (i) 125 mA; (ii) 250 mA.

8.4 Effect of Frequency. As shown in Example No. 6, when the frequency of the A.C. applied to a capacitor is increased, the capacitive reactance decreases and the current in the circuit increases correspondingly. The variations in reactance and current in a purely capacitive circuit when the frequency is varied, are as shown in Fig. 15a and b.



(a) Capacitive Reactance Decreases as Frequency Increases

(b) Current Increases as Frequency Increases

FIG. 15. EFFECT OF FREQUENCY ON CAPACITIVE CIRCUIT.

8.5 Capacitors Connected in Series. When capacitors are connected in series, the total reactance is the sum of the reactances of the individual capacitors.

$$X_C = X_{C1} + X_{C2}$$

8.6 Capacitors Connected in Parallel. The joint reactance of capacitors connected in parallel is determined in a similar manner to that employed for parallel resistors.

$$\frac{1}{X_C} = \frac{1}{X_{C1}} + \frac{1}{X_{C2}}$$

8.7 Phase Relationship. We have seen that current flows in a capacitive circuit, only while the capacitor is charging or discharging, or, in other words, when the state of charge is changing. Maximum current flows when the charge is changing at the greatest rate. In A.C. circuits containing capacitance only, this instant occurs when the applied voltage is at zero value. In such circuits, the current leads the applied voltage by 90°.

Fig. 16a illustrates a circuit containing "pure" capacitance. An alternating sine wave voltage is applied and a current flows. The voltage maintained across the capacitor is shown in Fig. 16b.

As the charge on the capacitor is proportional to the voltage maintained across it, ($Q = CE$), the charge on the capacitor rises and falls in phase with this voltage, as shown in Fig. 16b.

At points 2 and 4 on Fig. 16b, the charge is momentarily steady. As current can flow only when the state of charge is changing, the current at these points is zero.

At points 1, 3 and 5, the charge is changing at the greatest rate, and thus the current is at a maximum value at these instants. At points 1 and 5, the charge is changing in a positive direction, so current at these points reaches its maximum positive value. At point 3, the charge is changing in a negative direction therefore the current attains its maximum negative value. The variations in current are shown in Fig. 16c.

In Fig. 16d the current wave, which, for sine wave applied voltage, is also a sine wave, is referred to the voltage wave. We see that the current reaches its maximum positive value, one quarter of a cycle, or 90° before the voltage wave does so.

In a purely capacitive circuit therefore, the current leads the applied voltage by 90°. This is shown vectorially in Fig. 16e. When the circuit contains resistance as well as capacitance however, the phase difference between current and applied voltage is less than 90° as only portion of the applied voltage is dropped across the capacitive reactance.

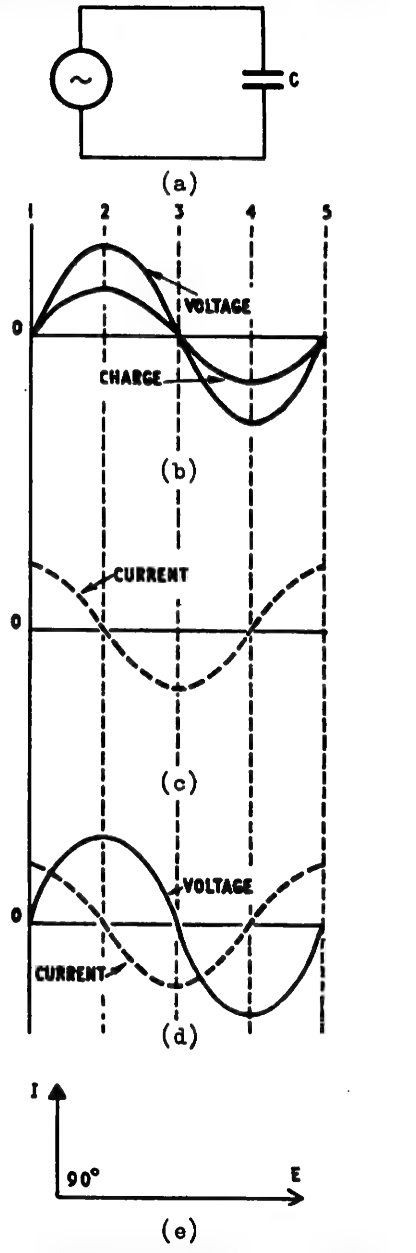


FIG. 16. CAPACITANCE IN A.C. CIRCUIT.

8.8 Power in Capacitive Circuits. A capacitor in an A.C. circuit charges and discharges twice during each cycle. The energy stored when the capacitor charges is returned to the circuit during discharge, and total power dissipation for each half cycle is zero.

In practice, a small amount of energy is converted to heat due to the resistance of the plates but, as this is small, it is often ignored.

8.9 Capacitance in Telecom. Circuits. The capacitor is a very important item in telecom circuits, and it would be a very difficult task to find one item of equipment that does not contain at least one capacitor. Some typical applications of capacitors in telecom are as follows -

(i) Speech Current By-pass Circuit. It is common practice in telecom circuits to include supervisory relays or sometimes non inductive resistors in the transmission circuits of C.B. P.M.B.Xs. and intercom. units. As these relays are inductive, their high opposition to A.C. speech currents could seriously reduce the level of transmission.

By connecting a capacitor (usually 2 μF) in parallel with the relay or resistor, as in Fig. 17, a comparatively low reactance path (40 ohms at 2 kc/s) is provided for speech currents.

FIG. 17. SPEECH CURRENT BY-PASS CIRCUITS.

(ii) Filter Circuits. We have seen that an inductor connected in series with the output of a rectifier smoothes out the pulsating D.C. However, even when the inductor is included, there is still an appreciable A.C. "ripple" present in the output. Should the rectifier be supplying power for the operation of a P.M.B.X. or similar equipment, a "hum" would be heard in every telephone.

This can be almost eliminated by connecting a capacitor across the output of the rectifier as in Fig. 18. The capacitor provides a low reactance path for this A.C. so that it does not pass through the load circuit.

The capacitor used must have a high value of capacitance so that its reactance to the low (power) frequency is small. Capacitors up to 40,000, μF are used in this application.

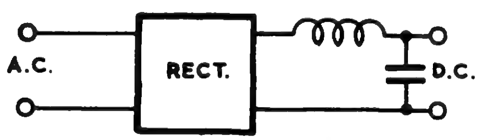


FIG. 18. FILTER CAPACITOR.

(iii) Coupling. Many telecom circuits require one section of the circuit isolated from another section as far as D.C. is concerned, but, at the same time, an A.C. connection must be provided between the two parts. This "coupling" can readily be provided by a capacitor. We find many examples of this in telecom, some of which are -

- the connection of the operators circuit in a C.B. switchboard cord circuit
- the Stone method of transmission feed.

There are many other applications of capacitance in our telecom equipment, but it is not possible to cover them all here. As a matter of interest, items of equipment, in which the capacitor is a vital component include -

amplifiers, oscillators, filters, equalisers, timing circuits, spark quench circuits ana voltage doubling rectifiers. These are covered in other papers of the course.

9. ADDITION OF OUT-OF-PHASE VOLTAGES OR CURRENTS.

9.1 In D.C. circuits, the resultant of a number of voltages or currents can be found by simple arithmetic. In A.C. circuits, the same method can be applied to "in phase" voltages and currents, but when currents or voltages are out of phase, they cannot be combined so simply.

There are two basic methods of adding "out of phase" alternating currents or voltages -

(i) the graphical method
(ii) the vectorial method.

9.2 Graphical Addition. Figs. 19 and 20 illustrate the usual method adopted for graphical addition.

Two sine wave voltages "A" and "B", 90° out of phase, each of 70 volts (peak value 100 volts approx.) are to be added. The procedure is -

(i) Draw graphs of the two voltages in their correct phase relationship along a common time axis, as is partly done in Fig. 19a.
(ii) Plot the direction and instantaneous values of one wave at a number of points (Fig. 19b).
(iii) Add these instantaneous values algebraically at corresponding points on the other wave (Fig. 19c).
(iv) Join the ends of these lines to form the graph of the resultant (Fig. 19c).

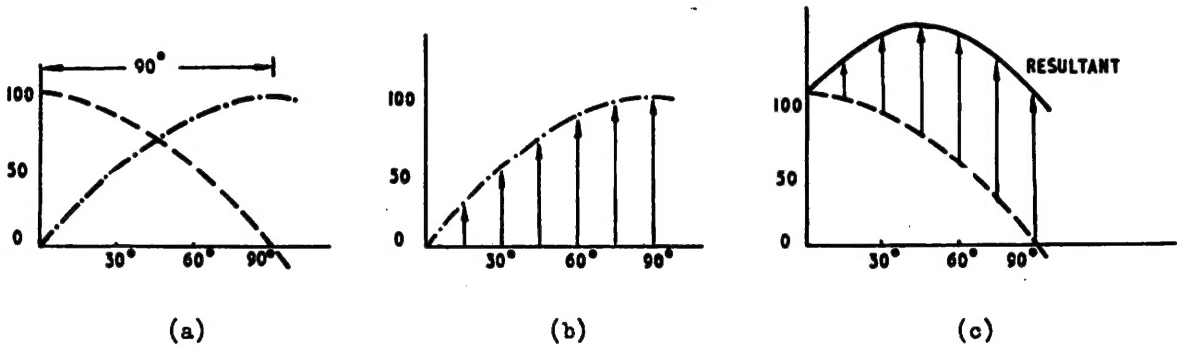


FIG. 19. GRAPHICAL METHOD.

Fig. 20 shows the completed diagram. The resultant has an effective value of approx. 100 volts (peak value 140 volts approx.) and there is a phase difference of 45° between it and each of the original waves.

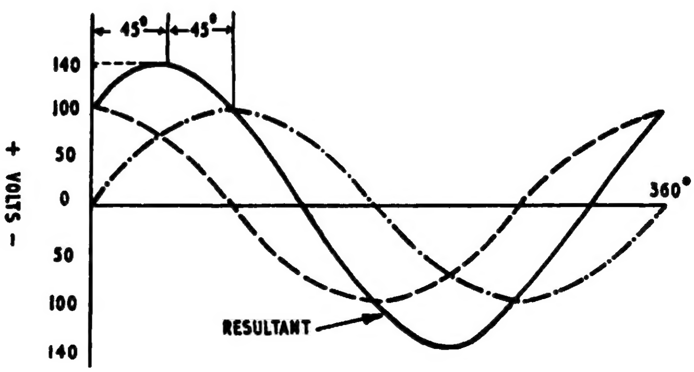


FIG. 20. GRAPHICAL ADDITION.

This method is also applicable to the addition of out of phase currents and is often used to show the wave form of two or more non sine waves.

9.3 Vector Addition. Where the wave form of both waves is the same, the resultant of two out of phase currents or voltages can be found quickly and accurately by using vectors.

A vector is a line whose length and direction represent the magnitude and direction of some physical quantity.

Vectors representing alternating currents and voltages are drawn so that -

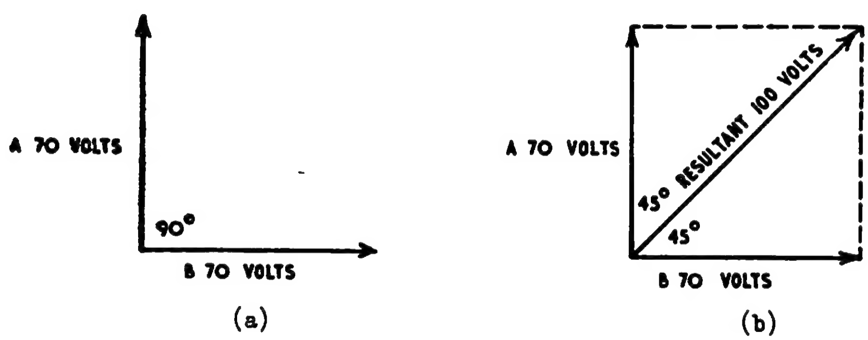
- the length of the vector represents the effective value.
- the angle at which the vectors are drawn to each other or to a reference line indicates the phase angle.
- the direction of rotation (anti-clockwise), indicates leading or lagging phase relationships.

It is customary to use open arrow-heads for voltage vectors and closed arrow-heads for current vectors.

Fig. 21 shows the method of combining the voltages of para. 9.2 by vectors. The procedure is :-

(i) Draw vectors representing the voltages "A" and "B" to scale. As voltage "A" leads voltage "B" by 90°, it is drawn at right angles above the "B" vector, (Fig. 21a).

(ii) Complete the parallelogram on vectors "A" and "B" (Fig. 21b). The length of the diagonal drawn from the point of intersection of the vectors represents the effective value of the resultant. The angle it makes with the original vectors indicates the phase relationship.



VECTOR ADDITION.

FIG. 21.

By measurement on Fig. 21b, the resultant has an effective value of 100 volts (approx.) and there is a 45° phase difference between it and the two original waves.

When a number of currents or voltages in the same circuit are to be added vectorially, it is convenient to show their phase relationship by drawing each in relation to a reference vector.

This reference vector must represent a factor in the circuit common to all the currents or voltages to be combined.

For example -

- voltages in a series circuit are referred to a common current reference vector.
- currents in a parallel circuit are referred to a common voltage reference vector.

It is customary to draw this reference vector horizontally.

9.4 Vector diagrams, similar to Fig. 21 are used extensively to illustrate A.C. circuit conditions and to simplify A.C. problems. Applications of vectors are given in the papers "Series A.C. Circuits" and "Parallel A.C. Circuits".

NOTES

10. TEST QUESTIONS.

1. What is an alternating current?

2. Frequencies used in telecom are measured in ...., ... or ....

3. Frequencies which produce sounds that can be heard are called ....

4. Radio Frequencies are those which ....

5. In trigonometry sine = ...
   cosine = ...
   tangent = ...

6. From the trigonometry tables,

   sin. $36^\circ 18'$ = ...; cos $42^\circ 10'$ = ...; tan $29^\circ 15'$ = ...

7. From the trigonometry tables,

   sin. ....°.....' is 0.6372; cos ....°.....' is 0.7026; tan ....°.....' is 0.5624.

8. The hypotenuse of a right-angle triangle is 41" long, and one side is 9" long. What is the length of the third side?

9. The angles included in this triangle would be ....° and ....°.

10. What is meant by "Phase angle"?

11. At 30° after the zero point, a sine wave A.C. has an instantaneous value of 5 amps. State -

    (i) The peak value ....
    (ii) The effective value ....

12. Alternating currents and voltages are measured in .... values.

13. What is meant by "form factor"?

14. In a purely resistive circuit, the current leads / lags / is in phase with the applied voltage.

15. At high frequencies, the resistance of a conductor increases / decreases due to ....

16. The opposition to A.C. offered by an inductor is called ....
    This opposition increases / decreases / remains unchanged when the frequency is increased.

17. In purely inductive circuits, current leads / lags / is in phase with voltage by ....°.

18. In purely capacitive circuits, the current is limited by the .... which increases / decreases / remains unchanged when the frequency is increased.

19. In such circuits, the current leads / lags / is in phase with the applied voltage by ....°.

20. What is a vector?

END OF PAPER